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Analytical Formulas for Disk Filters

Formulas relating the rate of filtrate flow in a sectioned disk filter to the process variables are derived. It is demonstrated that an optimum inner radius which yields a maximum flow of filtrate can be chosen. Flow rates for drum and disk filters operating under similar conditions are compared.

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SCOPE

Disk filters consist of a series of thin disks revolving on a common shaft and partially submerged in a slurry. They are useful where rapid cake buildup is possible under vacuum and washing is not required. Up to the present time, no mathematical analysis of the mechanism of a disk filter has been published. Empirical design has completely dominated the field.

A disk filter can be analyzed in a manner analogous to that used for a drum filter. However, the disk filter collects solids at variable distance from the axis of rotation, whereas all of the solids are deposited at a constant radius on the drum. An additional integration in the radial direction is necessary to obtain equations for the disc filter.

In the past, it has been customary (Nickolaus and Dahlstrom, 1956; Rushton et al. 1969; Tiller, 1972) to treat continuous rotary drums as though there were no internal sectioning and the vacuum were effective over the entire angle of submergence. Following the methodology of Risbud (1973), derivations in this paper account for sectioning and thus are more realistic than previous theoretical analyses of similar problems.

The objective of this study is to present formulas which can be used for the analysis and design of disk filters. The derivations are restricted to the case of constant specific filtration resistance and constant flow rate through the cake. While a more sophisticated analysis could have been made involving variable flow resistance and variable flow rate (Tiller and Shirato, 1964), the proposed equations represent a reasonable first step toward substituting theory for empiricism.

CONCLUSIONS AND SIGNIFICANCE

Design of disk filters has been entirely empirical in the past as no analytic equations have been available. A series of formulas which can be used for predicting behavior of disk filters under the limitation of constant specific resistance have been derived. (These formulas do not imply incompressible cakes.)

Equation (19) provides a double integral for numerical calculation of the flow rate as a function of number of sections, rotational speed, medium resistance, cake resistance, hydrostatic head, submergence angles, inner and outer radii of the disks, and slurry properties. Simplified formulas are presented for negligible medium resistance and hydrostatic head and also infinite sectioning.

Comparison of formulas for disk and drum filters indicate that the former may be expected to give two to three times the flow rate of the latter for equal floor space and the same radii.

Previously, it has not been recognized that there is an optimum value of the inner radius which yields a maximum flow rate. The optimum ratio of inner to outer radii varies from about 0.6 to 0.75 and is independent of the properties of the slurry.

DISK FILTERS

Disk filters are utilized in industry for materials, such as granulated coal, which filter with relative ease, lend themselves to continuous operations and do not require washing. Although processes requiring the three steps of precoating, washing, and drying can be more easily done on a drum or belt filter, drying can be accomplished by air or steam with disks. As illustrated in Figure 1, the filter consists of a series of disks A which rotate on a common axis in a slurry. The cake is scraped off at C and dropped into discharge chutes D. Each chute receives solids from facing sides of adjacent disks. The disks are divided into sections which have individual connections to the vacuum and air for the filtering, drying, and blow zones. Piping is enclosed in the central revolving barrel connected to a wear plate E which contacts the control block F. Both are shown in Figure 2. Filtration takes place on both sides of the disk. Vacuum is applied inside and the filter cake is deposited on the disk surface as it rotates through the slurry with only a part of the disk being submerged. At the end of the cycle, the vacuum is broken and the cake discharged. In most cases, the level of submergence is kept below the axis of the disk. Otherwise a trunnion stuffing box is required.

SECTIONING OF THE DISKS

Sectioning of the disk is necessary for operation of the separate filtration, drying, and blow portion of the cycle. Each section as shown in Figure 2 has its own independent connection to the control block. In addition, it is necessary to blank off a central area (radius R_1 in Figure 3) or else air would flow through the section between the center of rotation O and the slurry level. Constructing the disks with an inner radius R_1 results in the central circular area being unavailable for filtration. It will be shown later that the radius of this blank area can be optimized to give a maximum filtration rate.

Figure 3 shows one section of the disk filter as used in industrial practice. This section has filtration surface ABCD surrounded by a blank strip which acts as a dead area. There may be cake growth across the strip, particularly for easily filterable materials. With cake growth, part of the dead area becomes available for flow. Each of the N^* sections, including the dead area, subtends an angle of $2\pi/N$ radian or $360/N$ degrees.

The distance between the edge AB of the filter area and the edge EF of the next section is $2b$. Because the dividing strip has a constant width, the lines AB, CD, and EF are not radial. The lines AB and CD intersect at O′. The other two sides BC and AD are formed by arcs with center at O.

The angles BOC and AOD are $(2\pi/N - 2\sin^{-1}b/R_1)$ and $(2\pi/N - 2\sin^{-1}b/R_2)$, respectively. The width b of the dividing strip is roughly 1.9 cm while R_1 and R_2 range approximately from 0.3 to 1.8 and 0.9 to 2.4 m, respectively. In the range of R_1 to R_2 covered by the variable radius r, the ratio b/r is small and $\sin^{-1}b/r$ can be replaced by b/r. Although not negligible, b/R_1 and b/R_2 are small compared to $2\pi/N$.

The filter surface ABCD acts like a leaf in that growth of the cake can occur in the dead area. In such cases, the effective filtration area will be larger than ABCD. Shirato et al. (1964) developed theoretical and experimental equations for factors to correct the basic filter area for growth of two- and three-dimensional cakes deposited on leaves. For practical ranges of the ratio of cake thickness to leaf (rectangular or circular) dimension, it was shown that the correction factor was given by

$$j_3 = 1 + 1.47(L/d)^{1.20} \qquad (1)$$

where L is cake thickness and d is the radius of a circular leaf or half length of a rectangular leaf. For the disk dimensions previously stated and with cake thickness ranging from 12 to 35 mm, L/d would have values varying roughly from 0.01 to 0.15. Area growth could thus range from 1.5 to 20% of the base area. The dead area calculated as a 1.9-cm strip around the entire section amounts to about 5 to 20% of the total area. Thus in the case of an easily filterable material producing a thick cake, there would be substantial growth into the dead area.

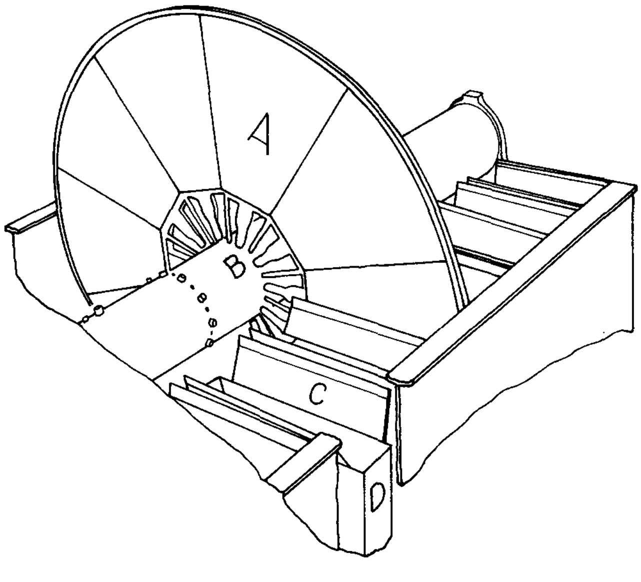

Fig. 1. Isometric view of disk filter.

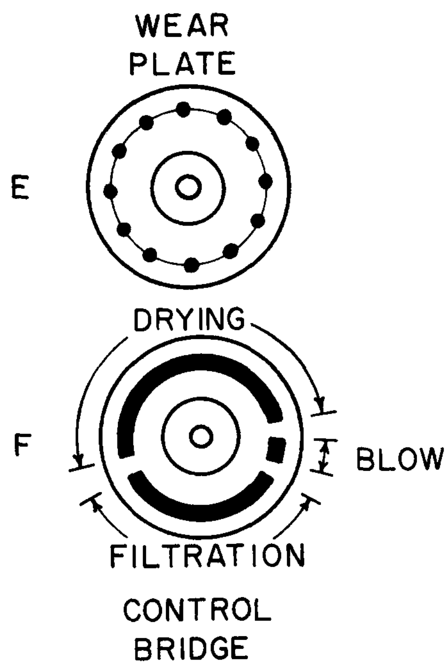

Fig. 2. Wearplate and bridgeblock.

* Agidisc filters manufactured by Eimco Division of Envirotech customarily utilize 8-12 sections (Eimco, 1964).

Contrariwise, a thin resistant cake would have minimal growth beyond the basic filtration area. In the derivation in this paper, the area ABCD will be used as the filtration area. The final results should be corrected for growth which can be substantial. The shape of a typical section differs from the rectangular and circular leafs employed by Shirato. Therefore, the Shirato area factor j_3 should be modified to fit the geometry of disk filters.

OPERATION OF THE DISK FILTER

Figure 4 displays an instantaneous view of the disk filter as it passes through a cycle. Section ABCD is partially submerged under the slurry.

Deposition of cake takes place on DEF due to hydrostatic head. Section GHIJ which is completely submerged is just being subjected to vacuum. Section KLMN has partially emerged from the slurry, and cake deposition has stopped on portion PLM which is completely out of the slurry.

Figure 5 shows section ABCD at different positions as the disk rotates through the slurry. In the first position, the section is just beginning to enter the slurry and is completely submerged at the second position. As the last point B_2 enters the slurry, the port through which the vacuum operates passes the bridge block, and the vacuum begins to be established. Silverblatt (1972) indicates that the vacuum is probably effective after 1/4 to 1/3 of the opening in the Wearplate passes the edge of the bridge block. The volume of the internal passages is relatively small, and the transient effect is quickly terminated. In this paper, it is assumed that the full vacuum is applied instantaneously.

The hydrostatic head at any point on the disk is due to the difference in heights of the slurry level and the point under consideration. If the filtrate is removed as it flows through the media, the entire head will be available. However, if the filtrate remains in a section until it emerges from the liquid, the hydrostatic head will be partially or completely neutralized. Therefore, hydrostatic head may or may not be effective in a rotary disk filter.

DERIVATION OF EQUATIONS

In Figure 5, section ABCD is shown in its initial position as it is about to enter the slurry. The authors apologize for the intricacy of the figure. The large number of angles required in the derivation must be shown on the same diagram. Each point on ABCD undergoes a different set of conditions during the filtration cycle.

There are three separate stages of flow. In the first stage before the vacuum p_v is applied, flow due to hydrostatic head alone takes place. For example, gravity flow occurs when point A passes from D to A_2 and C from B_2 to C_2. In the second phase, the full vacuum is in effect until C reaches C_3. At that point, the vacuum may be reduced to p_{v1} for the dewatering cycle. If the cake does not crack, the full vacuum p_v may be maintained, in which case there would only be two stages in the cycle.

In the method employed, a flow equation is written for a differential area, $dA = r dr d\phi$. Then a triple integration is utilized to calculate the flow rate for a typical section. The first integration follows dA in its passage through the slurry and is broken up into three parts corresponding to the following:

1. Time during which gravity flow occurs from dA_1 to dA_2.
2. Form period under full vacuum p_v from position dA_2 to dA_3.
3. Period of reduced vacuum p_{v1} from dA_3 to dA_4.

The second integration is carried out at constant radius or across a section of the disk from E to F as shown in its initial position in Figure 5. The third integration involves the radius r as it ranges from R_1 to R_2.

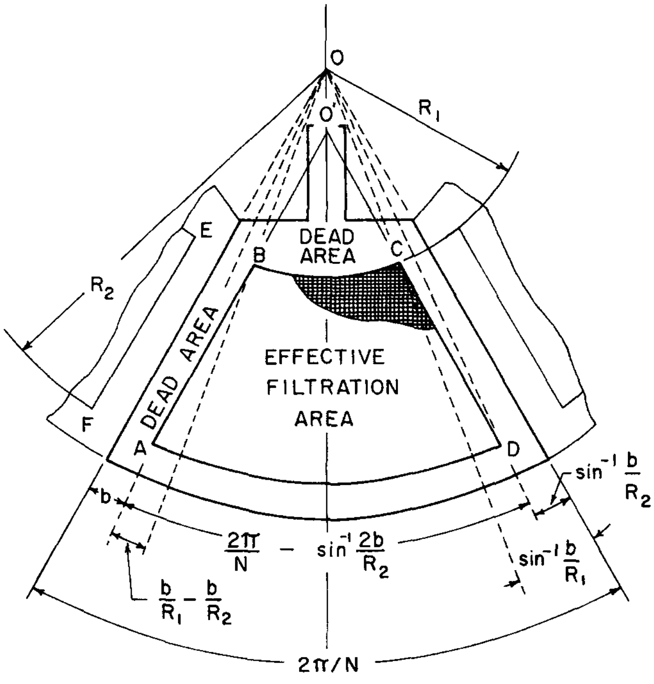

Fig. 3. Section of disk filter.

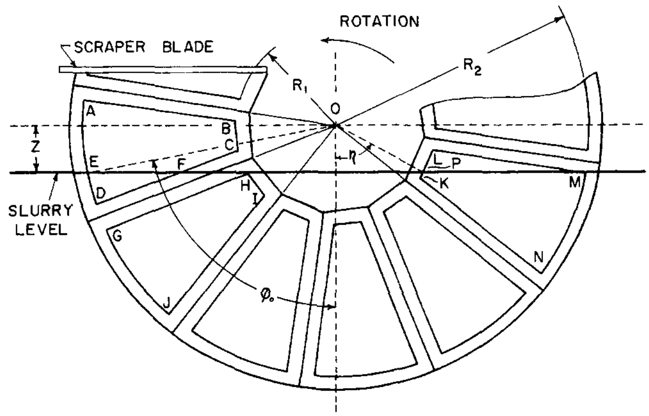

Fig. 4. Instantaneous view of disk filter.

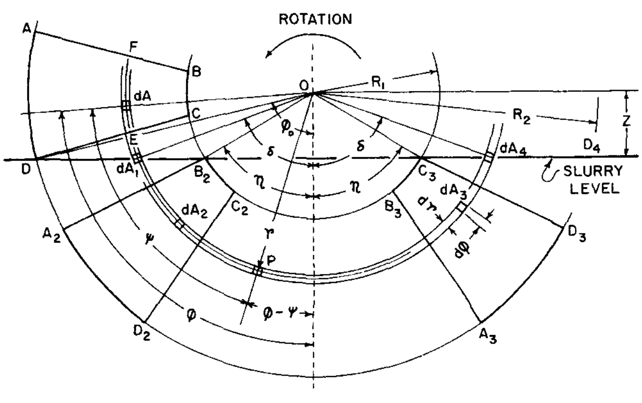

Fig. 5. Various positions of section ABCD.

The derivation will be limited to the case of constant permeability or specific filtration resistance α. Constant alpha implies either an incompressible cake or one filtered with a constant pressure drop across the cake. Limitations to these assumptions have been discussed by Tiller (1972) with respect to rotary drum units similar to disk filters and also with respect to constant pressure filtrations (Tiller, 1953, 1972). Errors of up to 20 to 25% might easily be encountered on the basis of the simplifications which have been employed.

The process is considered to originate at the moment D enters the slurry. From D to D_2, gravity flow occurs. When D reaches D_2, it is assumed that the vacuum p_v is instantaneously established; and then D_2 undergoes normal filtration until it reaches D_3. The section then begins to pass out of the slurry, and the vacuum is reduced to p_{v1}. Cake continues to build up from D_3 to D_4 at a reduced rate.

B is the only point which does not experience any flow due to gravity alone. As it revolves from B_2 to B_3, the full vacuum is in effect; while from B_3 to C_3, the vacuum is reduced to p_{v1}. Point C undergoes gravity flow from B_2 to C_2 and then is subjected to the maximum vacuum until it emerges at C_3.

A and D are submerged for the same length of time. Point A passes through arc DA_2 or angle $(\phi_0 - \eta - \sin^{-1}b/R_1 + \sin^{-1}b/R_2)$ while under hydrostatic flow. It then revolves through arc A_2A_3 or angle $(2\eta - 2\pi/N + 2b/R_1)$ with the full vacuum p_v. Finally it completes its cycle passing through arc A_3D_4 or angle $(\phi_0 - \eta + 2\pi/N - b/R_1 - b/R_2)$ under reduced vacuum p_{v1}.

An arbitrary element dA has initial coordinates (r, ϕ). As time proceeds, dA travels through an angle ψ to point P; and its coordinates become $(r, \phi - \psi)$. Initially $\psi = 0$. If the angular velocity is ω, then

$$\psi = \omega t \tag{2}$$

It is necessary to know the angles of locations of different points at every stage in order to be able to calculate the volume of filtrate per unit area. These angles are shown in Table 1.

FLOW EQUATION

If the specific resistance α is assumed constant, the basic flow equation can be written as

$$\frac{dv}{dt} = \frac{p}{\mu(\alpha w + R_m)} = \frac{p}{\mu(\alpha c v + R_m)} \tag{3}$$

where

$$c = \frac{s\rho}{1 - s/s_c} \tag{4}$$

and w is mass of dry solids/unit area, R_m is the medium resistance, and v is the volume of filtrate per unit area. The slurry concentration is given by s in mass fraction units or by c when expressed as mass of dry cake per unit volume of filtrate. The mass fraction of solids in the cake is s_c. The total pressure p equals the sum of the hydrostatic head and the vacuum. For element dA, the pressure between the four stages involving flow are

Stage no.	Description	Total pressure differential	
1 to 2	Gravity flow	$p = G = \rho_p g[r\cos(\phi - \psi) - R_2\cos\phi_0]$	(5)
2 to 3	Full vacuum	$p = G + p_v$	(6)
3 to 4	Partial vacuum	$p = G + p_{v1}$	(7)

The quantity G may be placed equal to zero if gravity flow is made ineffective by a counter hydrostatic head inside the disk. The density ρ_p of the prefilt will be greater than the density of the pure liquid and can be estimated by assuming that the volumes of liquid and solid are additive. A simple balance over unit mass of slurry yields

$$\frac{1}{\rho_p} = \frac{s}{\rho_s} + \frac{1-s}{\rho} \tag{8}$$

INTEGRATION OF FLOW EQUATION

Equation (3) can be rearranged and p can be substituted for by using Equations (5), (6), and (7) as follows:

$$\int_0^v \mu(\alpha c v + R_m)\, dv = \int_0^{t_1} (0)dt + \int_{t_1}^{t_2} G dt + \int_{t_2}^{t_3} (G + p_v)dt + \int_{t_3}^{t_4} (G + p_{v1})dt \tag{9}$$

The first integral from 0 to t_1 is shown with $p = 0$. The first integration is carried out over the time during which the differential element passes from dA to dA_1. No flow

TABLE 1. ANGLES SUBTENDED BY DIFFERENT POINTS WITH VERTICAL AT EVERY STAGE, RADIANS

Description	Stage No.	dA	A	B	C	D
Initial location	0	ϕ	$\phi_0 + \frac{2\pi}{N} - \frac{2b}{R_2}$	$\phi_0 + \frac{2\pi}{N} - \frac{b}{R_1} - \frac{b}{R_2}$	$\phi_0 + \frac{b}{R_1} - \frac{b}{R_2}$	ϕ_0
Element dA touches the slurry	1	δ	—	—	—	—
B touches the slurry	2	—	$\eta + \frac{b}{R_1} - \frac{b}{R_2}$	η	$\eta - \frac{2\pi}{N} + \frac{2b}{R_1}$	$\eta - \frac{2\pi}{N} + \frac{b}{R_1} + \frac{b}{R_2}$
C emerges from the slurry	3	—	$\eta - \frac{2\pi}{N} + \frac{b}{R_1} + \frac{b}{R_2}$	$\eta - \frac{2\pi}{N} + \frac{2b}{R_1}$	η	$\eta + \frac{b}{R_1} - \frac{b}{R_2}$
Element dA emerges from the slurry	4	δ	—	—	—	—

Where the angles $\eta = \cos^{-1}(R_2\cos\phi_0/R_1)$ and $\delta = \cos^{-1}(R_2\cos\phi_0/r)$. Since b/r is small, $\sin^{-1}(b/r)$ can be approximated by b/r.

takes place during that time. The decision to include an integral which is zero was based upon the desire to start all elements of ABCD at their original positions as shown in Figure 5 at the same time $t = 0$. While time could be started at the instant dA enters the slurry at position dA_1, the development is simplified by assuming a common reference angle.

The relationships between the angle ψ and the time t at the limits of the integrals in Equation (9) can be evaluated from Table 1 and are given by

$$\omega t_1 = \phi - \delta \quad (10)$$

$$\omega t_2 = \phi_0 + 2\pi/N - b/R_2 - b/R_1 - \eta \quad (11)$$

$$\omega t_3 = \phi_0 - b/R_2 + b/R_1 + \eta \quad (12)$$

$$\omega t_4 = \phi + \delta \quad (13)$$

where η and δ are as defined before. Substituting for G by using Equation (5), then substituting $\psi = \omega t$, integrating and substituting for time limits using Equations (10) to (13) yields

$$\mu\alpha c \frac{v^2}{2} + \mu R_m v = \frac{r\rho_p g}{\omega}[\sin(\phi - \omega t_1) - \sin(\phi - \omega t_4)]$$
$$- Z\rho_p g(t_4 - t_1) + p_v(t_3 - t_2) + p_{v1}(t_4 - t_3) \quad (14)$$

where

$$Z = R_2 \cos\phi_0 \quad (15)$$

Defining the right-hand side of Equation (14) as $F(r, \phi)$ where

$$\omega F(r, \phi) = 2r\rho_p g \sin\delta - 2Z\rho_p g\delta + 2p_v\left(\frac{b}{R_1} - \frac{\pi}{N} + \eta\right)$$
$$+ p_{v1}\left(\phi + \delta - \phi_0 + \frac{b}{R_2} - \frac{b}{R_1} - \eta\right) \quad (16)$$

it then becomes possible to place v in the form

$$\alpha c v = -R_m + \sqrt{R_m^2 + 2\alpha c F(r, \phi)/\mu} \quad (17)$$

The plus sign of the square root term is chosen in order that v be positive. The variable v represents the volume of filtrate per unit area per revolution which flows through dA.

Two more integrations must be performed to obtain the total flow rate. The volume of filtrate per pass associated with dA is simply $v dA = v r dr d\phi$. In order to account for an entire filter section ABCD in the integration, ϕ is integrated from ϕ_1 to ϕ_2 and r from R_1 to R_2. Thus, the filtrate volume V_s per section per revolution is given by

$$V_s = \int_{R_1}^{R_2}\int_{\phi_1}^{\phi_2} v r d\phi dr \quad (18)$$

where

$$\phi_1 = \phi_0 - b/R_2 + b/r$$

$$\phi_2 = \phi_0 + 2\pi/N - b/R_2 - b/r$$

As the disks are two-sided, the number of actual filtration sections passing through the slurry per unit of time is $2N\omega/2\pi = N\omega/\pi$. Multiplying V_s by $N\omega/\pi$ yields the total flow rate per disk. Substituting v from (17) in (18) yields the basic expression for the flow rate per disk, thus

$$Q = \frac{N\omega}{\pi}\int_{R_1}^{R_2}\int_{\phi_1}^{\phi_2}\left(-\frac{R_m}{c\alpha}\right.$$
$$\left. + \sqrt{\left(\frac{R_m}{c\alpha}\right)^2 + \frac{2F}{\mu c\alpha}}\right) r d\phi dr \quad (19)$$

Because of the complexity of F, it does not appear feasible to find a general analytical formula resulting from a direct process of integration.

SIMPLIFIED FORMULA

If the medium resistance and hydrostatic head variation are neglected and it is assumed that there is no change in vacuum, Equation (19) can be integrated. Placing $G = 0$ and $p_{v1} = p_v$ in (16) leads to the following:

$$\omega F(r, \phi) = p_v\left(-\frac{2\pi}{N} + \frac{b}{R_1} + \frac{b}{R_2} + \eta - \phi_0\right)$$
$$+ p_v(\phi + \delta) \quad (20)$$

The first term is independent of r and ϕ, and variation in the second term is small. For approximate calculation, the arithmetical average of the second term is taken over the points A, B, C, D

$$(\phi + \delta)_{av} = \left[\phi + \cos^{-1}\left(\frac{R_2}{r}\cos\phi_0\right)\right]_{av}$$
$$= \frac{1}{4}\left[\left(\phi_0 + \frac{2\pi}{N} - \frac{2b}{R_2} + \phi_0\right)\right.$$
$$+ \left(\phi_0 + \frac{2\pi}{N} - \frac{b}{R_2} - \frac{b}{R_1} + \cos^{-1}\left(\frac{R_2}{R_1}\cos\phi_0\right)\right)$$
$$\left. + \left(\phi_0 - \frac{b}{R_2} + \frac{b}{R_1} + \cos^{-1}\left(\frac{R_2}{R_1}\cos\phi_0\right) + 2\phi_0\right)\right] \quad (21)$$
$$= \frac{3\phi_0}{2} + \frac{\pi}{N} - \frac{b}{R_2} + \frac{1}{2}\cos^{-1}\left(\frac{R_2}{R_1}\cos\phi_0\right) \quad (22)$$

Substituting in $F(r, \phi)$

$$F(r, \phi) = \frac{p_v}{\omega}\left(\frac{\phi_0}{2} - \frac{\pi}{N} + \frac{b}{R_1} + \frac{3}{2}\eta\right) \quad (23)$$

Substituting for $F(r, \phi)$ and placing $R_m = 0$ in Equation (19) yields

$$Q = \frac{N}{\pi}\sqrt{\frac{2\omega p_v}{\mu c\alpha}}\int_{R_1}^{R_2}\int_{\phi_1}^{\phi_2}\left(\frac{b}{R_1} + \frac{\phi_0}{2}\right.$$
$$\left. + \frac{3}{2}\eta - \frac{\pi}{N}\right)^{1/2} r d\phi dr \quad (24)$$

Integrating with respect to ϕ and then r gives

$$Q = \left[R_2^2 - R_1^2 - \frac{2bN}{\pi}(R_2 - R_1)\right]$$
$$\sqrt{\frac{2\omega p_v}{\mu\alpha c}\left(\frac{\phi_0}{2} + \frac{3}{2}\eta + \frac{b}{R_1} - \frac{\pi}{N}\right)} \quad (25)$$

The maximum value of N occurs when points B and C (Figures 2 to 4) overlap, and $N = \pi R_1/b$. Thus, it is not possible to let N approach infinity in (25) without first letting b become zero. If $b = 0$ and N approaches infinity, Equation (25) becomes

$$Q = (R_2^2 - R_1^2)\sqrt{\frac{\omega p_v}{\mu\alpha c}(\phi_0 + 3\eta)} \quad (26)$$

Equation (19), (25), and (26) are all subject to the assumption of constant specific filtration resistance. Where compressible solids are involved, a value of α obtained from the average pressure as given by Equation (5) or (6) could be used. As cakes are generally compressed irreversibly, the value of α during the period of reduced vacuum would retain the magnitude reached during the period of full vacuum. In practical circumstances, it is necessary to use experimental data for each new material. The meager data appearing in the literature cannot be used directly because of variations in particle characteristics.

COMPARISON OF DISK AND DRUM FILTERS

It is useful to compare (26) with similar formulas for rotary drum filtration. Risbud (1974) developed the following approximate formula for a drum of radius R_2, width B, and divided into N sections

$$Q_R = R_2 B\left(1 - \frac{bN}{2\pi R_2}\right) \sqrt{\frac{2\omega p_v}{\mu \alpha c}\left(2\phi_0 - \frac{\pi}{N} + \frac{b}{2R_2}\right)} \quad (27)$$

where b is the width of the dead strip between sections. The medium resistance and hydrostatic head were neglected in developing (27) which is comparable to (25). If N approaches infinity and b is placed equal to zero, (27) reduces to the usual formula encountered in the literature. It is of interest to compare the rates of flow of a series of disk filters which occupy the same physical space as a drum and have the same radius R_2. Suppose that the distance between centers of disks is T, then the number of disks in the width B of the drum is B/T. Multiplying Q of Equation (26) by B/T and dividing by Q_R of (27) with infinite N and b equal to zero yields

$$\frac{Q}{Q_R} = \frac{R_2}{T}\left[1 - \left(\frac{R_1}{R_2}\right)^2\right]\sqrt{\frac{\phi_0 + 3\eta}{4\phi_0}} \quad (28)$$

Assuming that $\phi_0 + 3\eta$ is approximately equal to $4\phi_0$, Q/Q_R becomes a function of R_2/T, and R_2/R_1.

In the next section, it will be shown that for an angle of submergence of $2\phi_0 = 120°$, the optimum ratio for R_1/R_2 is about 0.6 whereas at 90° the optimum is about 0.75. In these two cases, Q/Q_R is then roughly equal to

$$Q/Q_R = 0.65\, R_2/T \quad 2\phi_0 = 120° \quad (29)$$

$$Q/Q_R = 0.4\, R_2/T \quad 2\phi_0 = 90° \quad (30)$$

With $T = 45.7$ cm (18 in.), and $R_2 = 1.83$ m (6 ft. drum), $R_2/T = 4.0$. Then the flow rate per unit of space occupied by the filter units will be approximately two to three times greater for the disk filters.

OPTIMIZATION OF INNER RADIUS R_1

It is possible to find an inner radius R_1 which will optimize the flow rate. If R_1 is so small that a part of the filtration surface is always above the slurry, the vacuum will be ineffective; and only gravity flow would take place. If R_1 is increased until it equals R_2, the filter area drops to zero; and no filtration takes place. Between these two extremes, there will be a R_2 which optimizes the flow rate. A simplified model will be investigated in which G and R_m are assumed negligible.

Defining the quantity

$$Y = Q/\sqrt{2\omega p_v/\mu\alpha c} \quad (31)$$

Equations (25) and (31) yield

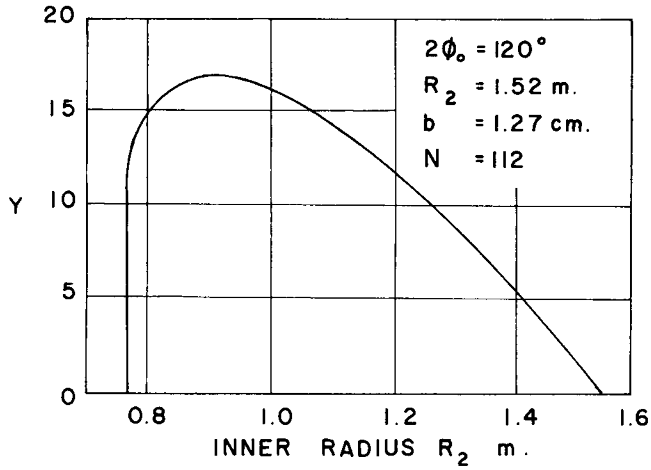

Fig. 6. Optimization of inner radius of a disk filter.

$$Y = \left[\left(R_2^2 - R_1^2\right) - \frac{2bN}{\pi}(R_2 - R_1)\right] \sqrt{\frac{3}{2}\cos^{-1}\left(\frac{R_2}{R_1}\cos\phi_0\right) + \frac{\phi_0}{2} - \frac{\pi}{N} + \frac{b}{R_1}} \quad (32)$$

Differentiation of Y with respect to R_1 produces an unwieldy expression, and it is better to use a numerical approach. It should be noted that the optimum value of Y is independent of the properties of the slurry. In Figure 6, a specific example is given for a disk with $R_2 = 1.52$ m (5 ft.), $N = 12$, $b = 1.52$ mm (0.05 ft.), and the half angle of submergence $\phi_0 = 60°$. The distance from the axis to the slurry is $Z = R_2 \cos\phi_0 = 0.76$ m. It can be seen that R_1 (optimum) is about 91 cm. Utilizing the factor f as defined by

$$f = \frac{R_1(\text{opt}) - R_2(\text{min})}{R_2 - R_2(\text{min})} = \frac{R_1(\text{opt}) - R_2\cos\phi_0}{R_2 - R_2\cos\phi_0} \quad (33)$$

it was found that f ranged from 0.17 to 0.2 for a series of problems in which R ranged from 60 to 240 cm (2 to 8 ft.), ϕ_0 from 40 to 80°, and N from 8 to 12. Solving for $R_1(\text{opt})/R_2$ yields

$$R_1(\text{opt})/R_2 = f + (1 - f)\cos\phi_0 \quad (34)$$

The ratio $R_1(\text{opt})/R_2$ varies from roughly 0.6 for $2\phi_0 = 120°$ to about 0.75 at $2\phi_0 = 90°$. At the higher submergence angles above 120°, the fraction of the area of the disk available for filtration is about 85 to 90% $((R_2^2 - R_1^2)/R_2^2)$ under optimum conditions. As the angle drops below 120°, the area corresponding to the optimum R_1 drops off rapidly and reaches approximately 0.5 at 80°.

When R_1 has its minimum value (78.5 cm in this example), the section ABCD does not become completely submerged until it reaches the 6 o'clock position. Deposition of solids continues after the section leaves that position because the cake which was deposited (assuming a sufficient thickness) protects the vacuum. However, if R_1 is less than 78.5 cm, the section will never be completely submerged, and the flow due to the vacuum will be zero. Thus, there is a discontinuity in the flow rate at the minimum R_1 with a sudden drop to zero.

CAKE THICKNESS

The cake thickness varies across the face of each section of the disk. It can be obtained by writing an equation relating v to L, thus,

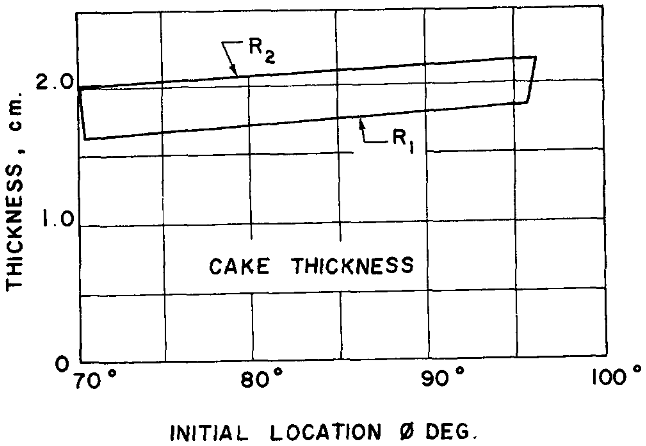

Fig. 7. Cake thickness distribution.

$$w = cv = \rho_s(1 - \epsilon_{av})L \qquad (35)$$

where L is cake thickness and ϵ_{av} is the average porosity of the cake. The expression for v in Equation (17) where $F(r, \phi)$ is given the appropriate values can be substituted in (35) to produce a relation for L as a function of r, ϕ, and t.

A specific example was worked out with the following conditions, and the results are plotted in Figure 7.

Material: Talc

α	$= 6.71(10^{10})$ m/kg	μ	$= 1.0$ mN s/m^2
R_m	$= 1.64(10^{10})$ m^{-1}	s	$= 0.2$ mass fraction
ρ	$= 1000$ kg/m^3	s_c	$= 0.32$ mass fraction
ρ_s	$= 2670$ kg/m^3	R_1	$= 0.726$ m (2.382 ft.)
ϕ_0	$= 60°$	R_2	$= 1.53$ m (5 ft.)
N	$= 16$	b	$= 0.0127$ m (0.5 in.)
ω	$= 0.0262$ rad./s	p_v	$= 68.947$ kN/m^2 (10 lb./sq.in.)

Minimum and maximum submergence times are experienced respectively by points C and A. Based upon the stated conditions, the ratio of maximum to minimum thickness is 1.35.

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NOTATION

b = width of blank strip around each section, m
B = drum width, M
c = mass of dry solids per unit volume of filtrate, kg/m^3
d = radius of a leaf or half diameter of a plate, m
f = ratio defined by Equation (33)
F = pressure at an arbitrary point, Equation (16), N/m^2
g = acceleration of gravity, 9.81 m/s^2
G = hydrostatic head, N/m^2
j_3 = area growth correction factor, Equation (1)
L = cake thickness, m
N = number of sections
p = total pressure differential, N/m^2
p_v = maximum vacuum, N/m^2
p_{v1} = reduced vacuum, N/m^2
Q = rate of filtrate flow per disk, m^3/s
Q_R = rate of filtrate flow for a drum, m^3/s
r = variable radius, m
R_1 = inner radius, m
R_2 = outer radius, m
s = mass fraction of solids in slurry
s_c = mass fraction of solids in cake
t = time, s
T = spacing between disks, m
v = volume of filtrate for a typical element dA per unit area per revolution, m^3/m^2 rev
V_s = volume of filtrate per section per revolution, m^3/rev
w = mass of dry solids per unit area kg/m^2
Y = defined by Equation (32)
Z = distance between axis of rotation and slurry level, m

Greek Letters

α = average specific filtration resistance, m kg^{-1}
δ = angle at which dA enters slurry, Table 1, radians
ϵ_{av} = average porosity of the filtrate cake
η = angle at which point B enters slurry, Table 1, radians
μ = liquid viscosity, kg/m s
ρ = density of pure liquid, kg/m^3
ρ_p = density of slurry, kg/m^3
ρ_s = density of solids, kg/m^3
ϕ = angle of initial location of dA, Figure 4, radians
ϕ_0 = angle of submergence, Figure 4, radians
ϕ_1 = $\phi_0 - b/R_2 + b/r$
ϕ_2 = $\phi_0 + 2\pi/N - b/R_2 - b/r$
ψ = angle traversed by arbitrary element dA
ω = angular velocity of the disk radians/s

Subscripts

1, 2, 3 indicate time at different stages of the filtration cycle

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